Hand your completed quiz in before the due date. Do not forget to write down your name and student ID number. Marks will be awarded for this quiz based on the clarity of your answers. The marker will pay close attention to the logic of your answers. Please show all your working.

Q1.
(a) Sketch the graph of a function that has a local maximum at 2 and is differentiable at 2.
(b) Sketch the graph of a function that has a local maximum at 2 and is continuous but not differentiable at 2.
(c) Sketch the graph of a function that has a local maximum at 2 and is not continuous at 2.

Q2. Find an example (formula) of a function defined on $[-1, 2]$ that has an absolute maximum but no absolute minimum.

Q3. Find the absolute maximum and absolute minimum values of $f$ on the given interval.
(a) $f(x) = 12 + 4 - x^2$, $[0, 5]$.
(b) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$.
(c) $f(x) = (x^2 - 1)^3$, $[-1, 2]$.
(d) $f(t) = t\sqrt{4 - t^2}$, $[-1, 2]$.
(e) $f(x) = \frac{x}{x^2 - 1}$, $[0, 3]$.
(f) $f(x) = 2 \cos x + \sin 2x$, $[0, \pi/2]$.

Q4. If $a$ and $b$ are positive numbers, find the maximum value of $f(x) = x^a(1 - x)^b$, $0 \leq x \leq 1$.

Q5. Prove that the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum nor a local minimum.

Q6. Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number $c$ in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle’s theorem?

Q7. Let $f(x) = 2 - |2x - 1|$. Show that there is no number $c$ such that $f(3) - f(0) = f'(c)(3 - 0)$. Does this contradict the Mean Value Theorem?

Q8. Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

Q9. Does there exist a function $f$ such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all $x \in \mathbb{R}$?

Q10. (optional) Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed. [Hint: If $f(t)$ represents the position of a runner at time $t$, his/her speed is given by $f'(t)$].