Hand your completed quiz in before the due date. Do not forget to write down your name and student ID number. Marks will be awarded for this quiz based on the clarity of your answers. The marker will pay close attention to the logic of your answers. Please show all your working.

Q1. Find the domain and codomain of the transformation defined by the equations, and determine whether the transformation is linear.

(a) \[
\begin{align*}
    w_1 &= 3x_1 - 2x_2 + 4x_3 \\
    w_2 &= 5x_1 - 8x_2 + x_3
\end{align*}
\] (b) \[
\begin{align*}
    w_1 &= x_1^2 - 3x_2 + x_3 - 2x_4 \\
    w_2 &= 3x_1 - 4x_2 - x_3^2 + x_4
\end{align*}
\]

Q2. Find the standard matrix \([T]\) for the linear operator \(T\) defined by the formula.

(a) \(T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)\). 
(b) \(T(x_1, x_2) = (x_1, x_2)\).
(c) \(T(x_1, x_2, x_3) = (4x_1, 7x_2, -8x_3)\).

Q3. In each part, use the standard matrix for \(T\) to find \(T(x)\).

(a) \(T(x_1, x_2) = (3x_1 - x_2, 2x_1 + x_2); \ x = (-1, 4)\).
(b) \(T(x_1, x_2, x_3) = (x_1, x_2, x_3); \ x = (-1, 4, 1)\).

Q4. Use matrix multiplication to find the reflection of \((-1, 2)\) about

(a) the \(x\)-axis. 
(b) the \(y\)-axis. 
(c) the line \(y = x\).

Q5. Use matrix multiplication to find the reflection of \((2, -5, 3)\) about

(a) the \(xy\)-plane. 
(b) the \(xz\)-plane.

Q6. Use matrix multiplication to find the orthogonal projection of \((2, -5)\) about

(a) the \(x\)-axis. 
(b) the \(y\)-axis.

Q7. Use matrix multiplication to find the image of the vector \((3, -4)\) when it is rotated through the angle of

(a) \(\theta = 30^\circ\). 
(b) \(\theta = 90^\circ\).

Q8. Find the standard matrix for the stated composition of linear operators on \(\mathbb{R}\).

(a) A rotation of \(90^\circ\), followed by a reflection about the line \(y = x\).

(b) An orthogonal projection on the \(y\)-axis, followed by a contraction with factor \(k = 1/3\).

Q9. Determine whether \(T_1 \circ T_2 = T_2 \circ T_1\).

(a) \(T_1: \mathbb{R}^2 \to \mathbb{R}^2\) is the orthogonal projection on the \(x\)-axis, and \(T_2: \mathbb{R}^2 \to \mathbb{R}^2\) is the orthogonal projection on the \(y\)-axis.

(b) \(T_1: \mathbb{R}^2 \to \mathbb{R}^2\) is the orthogonal projection on the \(x\)-axis, and \(T_2: \mathbb{R}^2 \to \mathbb{R}^2\) is the rotation through an angle \(\theta\).