Hand your completed quiz in before the due date. Do not forget to write down your name and student ID number. Marks will be awarded for this quiz based on the clarity of your answers. The marker will pay close attention to the logic of your answers. Please show all your working.

Q1. Find a row operation that will restore the given elementary matrix to an identity matrix.

(a) \[ \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \]

(b) \[ \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix} \]

(c) \[ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]

Q2. Consider the matrices. \( A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} \), \( B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix} \), \( C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \end{bmatrix} \). Find elementary matrices \( E_1, E_2, E_3 \) (if possible) such that

(a) \( E_1 A = B \).

(b) \( E_2 B = A \).

(c) \( E_3 B = C \).

If such a matrix does not exist, explain why.

Q3. Find the inverse of the given matrix if the matrix is invertible. You can check your answer with R (Use \( \text{inv} \) function. Do not forget to load \texttt{pracma} package.)

(a) \[ \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \]

(b) \[ \begin{bmatrix} -3 & 6 \\ 4 & 5 \end{bmatrix} \]

(c) \[ \begin{bmatrix} 6 & -4 \\ -3 & 2 \end{bmatrix} \]

(d) \[ \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix} \]

(e) \[ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \]

(f) \[ \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \end{bmatrix} \]

Q4. Consider the matrix \( A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix} \).

(a) Find elementary matrices \( E_1, E_2 \) such that \( E_2 E_1 A = I \).

(b) Write \( A^{-1} \) as a product of two elementary matrices.

(c) Write \( A \) as a product of two elementary matrices.

\[ \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \end{bmatrix} \]

Q5. Show that \[ \begin{bmatrix} 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix} \] is not invertible for any values of the entries.

Q6. Prove that if \( A \) is an invertible matrix and \( B \) is row equivalent to \( A \), then \( B \) is also invertible.