Course title: Multivariate calculus and differential equations

Lecturer: Tomohiro Uchiyama

Lecture time: Tuesday 14:50-16:20 & Thursday 13:05-14:35
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Office Hours: Tuesday 10:45-12:15 & Wednesday 10:45-12:15
Course webpage: https://tomohirouchiyama.wordpress.com

Course Description:

This course deals with techniques in multivariable calculus and differential equations which have applications in many areas of science, commerce and engineering. It is also preparation for many courses in advanced mathematics.

Topics covered: geometry of multivariable functions, partial derivatives, linearisation, multivariate chain rule, implicit function theorem; multivariate optimisation, sufficient conditions for optimality, Lagrange multipliers for optimisation problems; double/triple integrals, polar coordinates, Jacobian determinants, parametric curves, tangent vectors, line integrals, Green’s Theorem; Ordinary differential equations, separation of variables, variation of parameters, initial value problems, direction fields, systems of linear and nonlinear first order differential equations, Laplace transforms, Fourier series. Introduction to numerical methods.

Textbooks:


Course Aims:

A student who successfully completes this course will

1. be proficient in the basic techniques of multivariate calculus: partial derivatives, chain rule, Lagrange multipliers for optimisation problems, multivariate integration (in several coordinate systems), evaluation of line integrals.
2. apply their understanding of multivariate geometry to express and solve vector calculus problems using suitable notation and theorems.
3. be able to use standard techniques to solve linear differential equations.
4. be able to make an appropriate choice of analytical, geometric or numerical algorithm for a given problem.
5. be able to use calculus methods to solve standard applied problems.
6. have developed problem solving skills as part of a team and as an individual.
7. have developed written and oral communication skills, emphasising the ability to explain what the mathematics means.

Assessment:
Quizzes (weekly) 60%
Final Exam 40%

Course Outline:

Week 1: Vectors and the geometry of space 1
Three-dimensional coordinate systems, Vectors, The dot product, The cross product, Equations of lines and planes.

Week 2: Vectors and the geometry of space 2
Cylinders and quadric surfaces, Vector functions and space curves, Arc length and curvature.

Week 3: Partial derivatives 1
Functions of several variables, Limits and continuity.

Week 4: Partial derivatives 2
Partial derivatives, Tangent planes and linear approximations, The chain rule.

Week 5: Partial derivatives 3
Directional derivatives and the gradient vector, Maximum and Minimum values, Lagrange multipliers.

Week 6: Multiple integrals 1
Double integrals over rectangles, Double integrals over general regions.

Week 7: Multiple integrals 2
Double integrals in polar coordinates, Triple integrals, Triple integrals in cylindrical coordinates.

Week 8: Multiple integrals 3
Change of variables in multiple integrals, Triple integrals in spherical coordinates.

Week 9: Vector calculus
Vector fields, Line integrals, Green’s theorem.

Week 10: Ordinary differential equations 1
ODEs, Separation of variables, Variation of parameters.

Week 11: Ordinary differential equations 2
Initial value problems, Direction fields, Systems of ODEs.

Week 12: Ordinary differential equations 3
Laplace transform, Fourier series.

Week 13: Numerical solutions for ODEs 1
Introduction to numerical solutions for ODEs.
Week 14: Numerical solutions for ODEs 2
Introduction to numerical solutions for ODEs.

Week 15: Final examination